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LETTER

Power diagram based algorithm for the facility location and capacity acquisition problem with dense demand

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1 Introduction

The facility location and capacity acquisition problem (LCAP) is essential to real-world applications, e.g., the cloudlet placement for mobile edge computing [1] and automotive service firms locating [2], which mainly involves the issues of where to build facilities, how much capacity to acquire, and which parts of the market region each facility should serve.

There are roughly two categories of methods to investigate the LCAP models. The first one is the discrete model, and the other one is the continuous model. For the former model, probability distribution functions, such as Poisson distribution and Normal distribution, are used to estimate the demands of customers. Different from the former model, dense demands, represented by the approximation continuous functions, are considered in the latter model.

Existing research algorithms have certain limitations for the LCAP with dense demand. Iri et al. [3] introduce a Voronoi diagram based algorithm for the LCAP. Murat et al. [4] propose a vertex-based iterative algorithm to solve the LCAP, which requires much more time consuming as compared with the previous algorithms. However, the allocation decisions are determined based on the Euclidean distance in the above algorithms, which cannot well express the attraction process in some real-world applications. For instance, the attraction of facilities is closely related to the scale and service level of facilities in the retail store locating. Bourne et al. [5] present an iterative algorithm for the LCAP with uniform dense demand. In addition to the distance, the attraction of facilities depends on a set of features, which are characterized by the weighting coefficients. However, due to the diverse demands of customers, these demands are often randomly distributed, so modeling demands as an approximation continuous function is more preferable in actual applications [4].

Therefore, we propose a power diagram based iterative method for the LCAP with dense demand, in which the gradient descent method is employed to update the weights of facilities, and Lloyd's method is applied to optimize the location of each facility. The main contributions of this work are summarized as follows:

1) introducing a continuous model for the LCAP, in which the transportation cost is a non-linear function of the Euclidean distance.
2) employing the power diagrams to determine the allocation decisions with general dense demand.
3) developing a novel iterative method to solve the LCAP with dense demand, which is fast and efficient.

2 Continuous model

Consider a market region $M \subset R^2$ in the convex polygon shape, over which a large number of customers are distributed, which are modeled as a spatial density function $\rho(\mathbf{x})$. Let $\mathbf{X} = \{\mathbf{x}_i\}_{i=1}^n$ denote the coordinates of n independent facilities, where $\mathbf{x}_i = (x_i, y_i) \in M$ represents the location of ith facility. The market region M is therefore partitioned into n disjoint subregions (called "service regions") $\mathbf{R} = \{R_i\}_{i=1}^n$, and service region R_i is assigned to the facility located at $\mathbf{x}_i$. Based on the previous studies on the LCAP, the total cost has three main components in our model: (1) the annualized fixed cost F_i, (2) the capacity acquisition cost f_i, and (3) the transportation cost C_i. A multi-objective optimization continuous model is built to find the optimal solution as follows:

$$\begin{cases} \min\limits_{\mathbf{X},\mathbf{R}} \quad E_{LCAP} = \sum_{i=1}^n \{F_i + f_i + C_i\}, \\ s.t. \quad F_i = F, \\ \qquad f_i = k + am_i + bm_i^{\alpha}, \\ \qquad C_i = c\int_{R_i} \rho(\mathbf{x}) \|\mathbf{x}_i - \mathbf{x}_j\|^2 d\mathbf{x}, \\ \qquad m_i = \int_{R_i} \rho(\mathbf{x}) d\mathbf{x}, \\ \qquad \sum_{i=1}^n m_i = \int_M \rho(\mathbf{x}) d\mathbf{x}, \end{cases} \tag{1}$$

where F, k, a, b, c are constant coefficients, and exponent α represents the economies (0< α <1) or diseconomies (α >1).

3 Proposed metohodology

3.1 Allocation method

There are some situations where the Euclidean distance cannot well express the attraction process of each facility, such as the retail store locating. In addition to the distance, the attraction of facilities depends on a set of features, which are charac-

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terized by the weighting coefficients $\mathbf{W}=\{w_i\}_{i=1}^n$. In real-world applications, the larger scale and the higher service level of facility, the greater the weighting coefficient, which means that more customers will be attracted, and the larger service region will be assigned to it.

As an extension of the Voronoi diagram, power diagrams [6] play an essential role in solving the LCAP. An important characteristic of power diagrams is that the resulting power partitions are always convex, and the bisectors are straight lines. Furthermore, power diagrams have the characteristics of precise capacity constraints. In this letter, the allocation decisions are determined based on the power diagrams as follows:

$$R_i=\{x\in M \mid \|\mathbf{x}-\mathbf{x}_i\|^2-w_i\leqslant\|\mathbf{x}-\mathbf{x}_j\|^2-w_j,\forall j\neq i\}. \tag{2}$$

Consequently, the strategic decisions based on the power diagram are actually the facility locations and weighting coefficients (or "weights") decisions.

3.2 Power diagram based iterative method

We apply the power diagrams to determine the allocation decisions in this letter, in which the attraction of facilities is characterized by the weighting coefficients. As soon as the location decisions and weighting coefficients are decided, the solutions can be easily obtained. In this letter, an effectively iterative method is introduced, where two key operations are: (1) finding the optimal weights $\mathbf{W}$ of facilities using the gradient descent method; and (2) employing Lloyd's method to optimize the location $\mathbf{X}$ of facilities.

3.2.1 Weight optimization

Given the location of each facility, the LCAP aims to find the weights of facilities to minimize the total cost in Eq. (1). We adopt the gradient descent method to update the weights of facilities, in which the backtracking line search method is applied to compute the optimal step size to minimize the total cost. The derivation of the total cost E_{LCAP} w.r.t., w_i of facility i can be computed as follows:

$$\begin{cases}\nabla_{w_i}E_{\mathrm{LCAP}}=\sum_{j\in\Omega_i}[\alpha b(m_j^{\alpha-1}-m_i^{\alpha-1})+c(w_j-w_i)]\nabla_{w_i}m_j,\\ \nabla_{w_i}m_j=-\dfrac{\bar{\rho}_{ij}}{2}\cdot\dfrac{|e_{ij}^*|}{|e_{ij}|},\end{cases} \tag{3}$$

where the average value of the field $\rho(\mathbf{x})$ over e_{ij}^* is referred as $\bar{\rho}_{ij}$.

3.2.2 Location optimization

Given a set of service regions, the LCAP can be reduced to the p-median problem, which locates a set of facilities to minimize the total cost between customers and facilities. Existing studies have been conducted to solve the p-median problem, and indicate that the transportation cost is minimized when each facility is located at the mass center of the corresponding service region [4]. In this letter, Lloyd's method is employed to optimize the location of each facility, that is, the location of facility i is moved to the mass center $\tilde{\mathbf{x}}_i$ of the service region R_i according to the following term:

$$\tilde{\mathbf{x}}_i=\frac{\int_{R_i}\mathbf{x}\rho(\mathbf{x})\mathrm{d}\mathbf{x}}{\int_{R_i}\rho(\mathbf{x})\mathrm{d}\mathbf{x}}. \tag{4}$$

Consequently, the power diagram based iterative algorithm for solving the LCAP is given as follows:

Algorithm 1 Iterative algorithm for the LCAP

Input: the market region M, demand density function $\rho(\mathbf{x})$, the number of facilities n,and a threshold ϵ as the termination
Output: $(\mathbf{X},\mathbf{R})$
Initialization: set $k=0$, $\mathbf{X}^{(0)}$ to be n randomly generated sites, and $\mathbf{W}=\mathbf{0}$
repeat
  Compute the gradient $\nabla_{w_i}E_{LCAP}$ in Eq. (3)
  Estimate the step-size Δt for updating $\mathbf{W}$ by the back-tracking line search method
  $w_i=w_i+\Delta t\cdot\nabla_{w_i}E_{LCAP}$
  Compute the mass centers $\tilde{\mathbf{x}}$ of service regions based on Eq. (4)
  Move the facilities to their respective mass centers using the Lloyd's method
  $k=k+1$
  Compute the total cost $E_{LCAP}^{(k)}$ in Eq. (1)
until $\frac{|E_{LCAP}^{(k)}-E_{LCAP}^{(k-1)}|}{E_{LCAP}^{(k-1)}}<\epsilon$

4 Performance evaluation

The performance of our proposed method is evaluated under various demand densities, including uniform (UD), linear (LD), and non-linear (NLD) demand densities, which are given in Eq. (5).

$$\rho(x,y)=\begin{cases}0.1+1.8x, & \text{LD-1},\\ 1.8-0.9x-0.9y, & \text{LD-2},\\ 0.1+x^2+y^2, & \text{NLD-1},\\ 0.1+2\times[(x-0.5)^2+(y-0.5)^2], & \text{NLD-2}.\end{cases} \tag{5}$$

Based on the description in [4,5], the market region M is selected as a squared service region with a side length of 1, and the coefficient b is set to 0.5, c is set to 100. Besides, the threshold value ϵ is fixed at 10^{-5} in our experiments.

As shown in Fig. 1, we plot the location of facilities in the market region along with the corresponding power partitions and depict the resulting layouts obtained by our proposed method. In this experiment, the demands of customers are modeled as a uniform dense demand. Besides, the number of facilities is selected from $\{2,3,\ldots,10\}$, and the exponent α is set to 0.5. From the results, we can observe that our proposed method can effectively solve the LCAP with uniform dense demand.

The final layout configurations for different demand densities are presented in Fig. 2, where the number of facilities is set to 12, and the exponent is 0.5. Four different demand densities are considered in this experiment, as shown in Eq. (5). The computational results prove the effectiveness of our proposed method for the LCAP with complex demand densities.

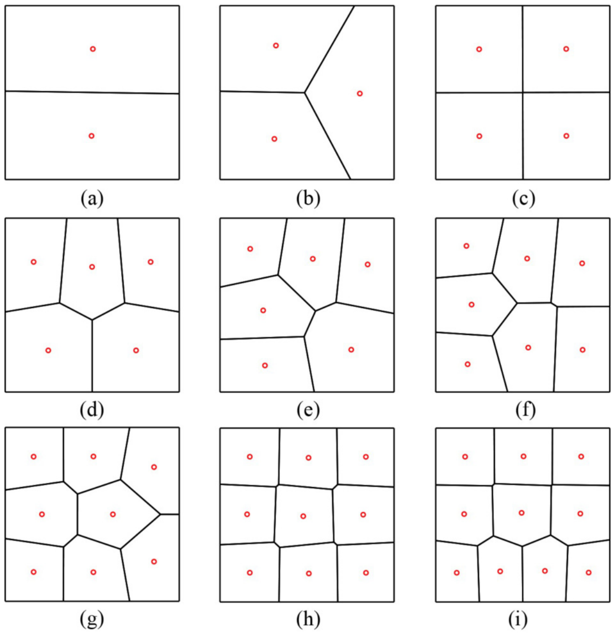

Fig. 1 Computational results for different numbers of facilities. (a) n = 2; (b) n = 3; (c) n = 4; (d) n = 5; (e) n = 6; (f) n = 7; (g) n = 8; (h) n = 9; (i) n = 10

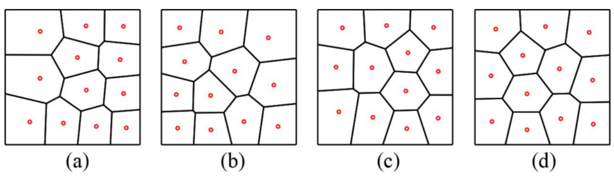

Fig. 2 Results for five facilities with various demand densities. (a) LD-1; (b) LD-2; (c) NLD-1; (d) NLD-2

As shown in Table 1, the computational time of the LCAP obtained by our proposed method and other methods are reported, where $\alpha = 0.5$ in our method, $a = 1.0$ in the method proposed by Murat [4], and $\lambda = 0.5$ in the method proposed by Bourne [5]. In this experiment, the squared region with a side length of 1 is selected as the default market region, the number of facilities is set to 16, and the threshold value ϵ is fixed at 10^{-5}. Besides, three demand densities are considered: uniform (UD), linear (LD-1), and non-linear (NLD-1) demand densities. From the results, we can observe that our proposed method outperforms other methods, which greatly improves the efficiency of solving the locating problems in operations research.

Table 1 Performance comparison to other methods with various demand densities (in seconds)

Methods	Allocation strategy		Demand densities		
	VD$_1$	PD$_2$	UD	LD-1	NLD-1
Iri et al. [3]	✓		4.230	4.536	4.457
Murat et al. [4]	✓		99.890	115.24	103.75
Bourne et al. [5]		✓	3.321	/	/
Proposed method		✓	2.198	2.199	2.538

Note: VD: Voronoi diagram, PD: power diagram

5 Conclusion

A novel continuous model for the LCAP with dense demand is proposed in this letter. Various demand densities are employed to adapt the population distribution in the market region, and a set of weighting coefficients are used to characterize the features of facilities, which represent the attracting process of facilities. Next, the power diagrams are applied to determine the allocation decisions of facilities, and a power diagram based iterative method is developed for solving the LCAP. Experimental results demonstrate the effectiveness of our proposed method.

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Supporting information The supporting information is available online at journal.hep.com.cn and link.springer.com.

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